

## TEST SPECIFICATIONS: SAT MATH TEST

# A Transparent Blueprint

This section describes the content, format, and distinctive new features of the Math Test in the redesigned SAT, as well as the skills it measures. This section also includes annotated sample questions that help illustrate central aspects of the test.

### OVERALL CLAIM FOR THE TEST

The redesigned SAT's Math Test is intended to collect evidence in support of the following claim about student performance:

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Students have fluency with, understanding of, and the ability to apply the mathematical concepts, skills, and practices that are most strongly prerequisite and central to their ability to progress through a range of college courses, career training, and career opportunities.

### TEST DESCRIPTION

In keeping with the evidence about essential requirements for college and career readiness described in Section II, the redesigned SAT will require a stronger command of fewer, more important topics. To succeed on the redesigned SAT, students will need to exhibit mathematical practices, such as problem solving and using appropriate tools strategically. The SAT will also provide opportunities for richer applied problems.

The redesigned SAT's Math Test has four content areas:

- » Heart of Algebra
- » Problem Solving and Data Analysis
- » Passport to Advanced Math

## » Additional Topics in Math

Questions in each content area span the full range of difficulty and address relevant practices, fluency, and conceptual understanding.

## Test Summary

The following table summarizes the key content dimensions of the redesigned SAT's Math Test.

SAT MATH TEST CONTENT SPECIFICATIONS		
<b>Time Allotted</b>	80 minutes	
Calculator Portion (38 questions)	55 minutes	
No-Calculator Portion (20 questions)	25 minutes	
	<b>NUMBER</b>	<b>PERCENTAGE OF TEST</b>
<b>Total Items</b>	58 questions	100%
Multiple Choice (MC, 4 options)	45 questions	78%
Student-Produced Response (SPR — grid-in)	13 questions	22%
<b>Contribution of Items to Subscores</b>		
<b>Heart of Algebra</b>	19 questions	33%
Analyzing and fluently solving equations and systems of equations		
Creating expressions, equations, and inequalities to represent relationships between quantities and to solve problems		
Rearranging and interpreting formulas		
<b>Problem Solving and Data Analysis</b>	17 questions	29%
Creating and analyzing relationships using ratios, proportions, percentages, and units		
Describing relationships shown graphically		
Summarizing qualitative and quantitative data		
<b>Passport to Advanced Math</b>	16 questions	28%
Rewriting expressions using their structure		
Creating, analyzing, and fluently solving quadratic and higher-order equations		

## SAT MATH TEST CONTENT SPECIFICATIONS

Manipulating polynomials purposefully to solve problems

<b>Additional Topics in Math*</b>	6 questions	10%
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Making area and volume calculations in context

Investigating lines, angles, triangles, and circles using theorems

Working with trigonometric functions

### Contribution of Items to Cross-Test Scores

Analysis in Science	8 questions	14%
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Analysis in History/Social Studies	8 questions	14%
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\*Questions under Additional Topics in Math contribute to the total Math Test score but do not contribute to a subscore within the Math Test.

The test covers all mathematical practices, with an emphasis on problem solving, modeling, using appropriate tools strategically, and looking for and making use of structure to do algebra. The practices emphasized in the redesigned SAT are central to the demands of postsecondary work. Problem solving requires students to make sense of problems and persevere to solve them, a skill highly rated by postsecondary instructors (Conley et al., *Reaching the Goal*, 2011). Modeling stresses applications characteristic of the entire postsecondary curriculum. Students will be asked throughout high school, college, and careers to make choices about which tools to use in solving problems. Finally, structure is fundamental to algebra and to other more advanced mathematics.

As indicated in the test specifications above, the Math Test has two portions. One is a 55-minute portion comprising 38 questions for which students are allowed to use calculators to solve the problems. The other is a 25-minute portion comprising 20 questions for which students are not allowed to use calculators to solve the problems. The blueprint for each of these portions is shown below.

**CALCULATOR PORTION**

	Number of Questions	% of Test
<b>Total Questions</b>	<b>38</b>	<b>100%</b>
Multiple Choice (MC)	30	79%
Student-Produced Response (SPR — grid-in)	8	21%
<b>Content Categories</b>	<b>38</b>	<b>100%</b>
Heart of Algebra	11	29%
Problem Solving and Data Analysis	17	45%
Passport to Advanced Math	7	18%
Additional Topics in Math	3	8%
<b>Time Allocated</b>	55 minutes	

**NO-CALCULATOR PORTION**

	Number of Questions	% of Test
<b>Total Questions</b>	<b>20</b>	<b>100%</b>
Multiple Choice (MC)	15	75%
Student-Produced Response (SPR — grid-in)	5	25%
<b>Content Categories</b>	<b>20</b>	<b>100%</b>
Heart of Algebra	8	40%
Passport to Advanced Math	9	45%
Additional Topics in Math	3	15%
<b>Time Allocated</b>	25 minutes	

## Detailed Description of the Content and Skills Measured by the SAT Math Test

The SAT has been redesigned to better align to what research shows students need to know and be able to do in order to be prepared for college and careers. This goal has led to a more focused SAT with a balance across fluency, conceptual understanding, and application. In these and other ways, such as embedding mathematical practices, the redesigned SAT is also a good reflection of college- and career-ready standards.

We will continue to be guided by research and evidence as we develop the redesigned SAT. In the months leading up to its release, for example, we may find through research that we need to adjust elements described in this document, such as time limits, number of questions or tasks, or scores reported. When and if we make these or other changes, we will do so solely to enhance the validity evidence supporting the test for its intended purposes, and we will communicate those changes as widely as possible and in a timely manner.

## HEART OF ALGEBRA: LINEAR EQUATIONS AND FUNCTIONS

SAT HEART OF ALGEBRA DOMAIN	
Content Dimension	Description
<b>Application</b>	
1. Create, solve, or interpret linear equations in one variable.	The student will create, solve, or interpret a linear expression or equation in one variable that represents a context. The expression or equation will have rational coefficients, and multiple steps may be required to simplify the expression, simplify the equation, or solve for the variable in the equation.
2. Create, solve, or interpret linear inequalities in one variable.	The student will create, solve, or interpret a linear inequality in one variable that represents a context. The inequality will have rational coefficients, and multiple steps may be required to simplify or solve for the variable.
3. Build a linear function that models a linear relationship between two quantities.	The student will describe a linear relationship that models a context using either an equation in two variables or function notation. The equation or function will have rational coefficients, and multiple steps may be required to build and simplify the equation or function.
4. Create, solve, and interpret systems of linear inequalities in two variables.	The student will analyze one or more constraints that exist between two variables by creating, solving, or interpreting an inequality in two variables or a system of inequalities in two variables to represent a context. Multiple steps may be required to create the inequality or system of inequalities or to determine whether a given point is in the solution set.
5. Create, solve, and interpret systems of two linear equations in two variables.	The student will analyze one or more constraints that exist between two variables by creating, solving, or analyzing a system of linear equations to represent a context. The equations will have rational coefficients, and multiple steps may be required to simplify or solve the system.
<b>Fluency</b>	
6. Solve linear equations in one variable.	The student will algebraically solve an equation (or inequality) in one variable. The equation (or inequality) will have rational coefficients and may require multiple steps to solve for the variable; the equation may yield no solution, one solution, or infinitely many solutions. The student may also be asked to determine the value of a constant or coefficient for an equation with no solution or infinitely many solutions.
7. Solve systems of two linear equations in two variables.	The student will algebraically solve a system of two linear equations in two variables. The equations will have rational coefficients, and the system may yield no solution, one solution, or infinitely many solutions. The student may also be asked to determine the value of a constant or coefficient of an equation in which the system has no solution, one solution, or infinitely many solutions.
<b>Conceptual Understanding</b>	
8. Interpret the variables and constants in expressions for linear functions within the context presented.	The student will make connections between a context and the linear equation that models the context and will identify or describe the real-life meaning of a constant term, a variable, or a feature of the given equation.

## SAT HEART OF ALGEBRA DOMAIN

Content Dimension	Description
9. Understand connections between algebraic and graphical representations.	The student will select a graph described by a given linear equation, select a linear equation that describes a given graph, determine the equation of a line given a verbal description of its graph, determine key features of the graph of a linear function from its equation, or determine how a graph may be impacted by a change in its equation.

Algebra is the language of much of high school mathematics, and it is also an important prerequisite for advanced mathematics and postsecondary education in many subjects. The redesigned SAT focuses strongly on algebra and recognizes in particular the essentials of the subject that are most essential for success in college and careers. Heart of Algebra will assess students' ability to analyze, fluently solve, and create linear equations and inequalities. Students will also be expected to analyze and fluently solve equations and systems of equations using multiple techniques.

To assess full command of the material, these problems will vary significantly in form and appearance. Problems may be straightforward fluency exercises or may pose challenges of strategy or understanding, such as interpreting the interplay between graphical and algebraic representations or solving as a process of reasoning. Students will be required to demonstrate both procedural skill and a deeper understanding of the concepts that undergird linear equations and functions to successfully exhibit a command of the Heart of Algebra.

Mastering linear equations and functions has clear benefits to students. The ability to use linear equations to model scenarios and to represent unknown quantities is powerful across the curriculum in the postsecondary classroom as well as in the workplace. Further, linear equations and functions remain the bedrock upon which much of advanced mathematics is built. Consider, for example, that derivatives in calculus are used to approximate curves by straight lines and to approximate nonlinear functions by linear ones. Without a strong foundation in the core of algebra, much of this advanced work remains inaccessible.

## PROBLEM SOLVING AND DATA ANALYSIS: PROPORTIONAL RELATIONSHIPS, PERCENTAGES, COMPLEX MEASUREMENTS, AND DATA INTERPRETATION AND SYNTHESIS

SAT PROBLEM SOLVING AND DATA ANALYSIS DOMAIN	
Content Dimension	Description
<b>Application</b>	
1. Use ratios, rates, proportional relationships, and scale drawings to solve single- and multistep problems.	The student will use a proportional relationship between two variables to solve a multistep problem to determine a ratio or rate; calculate a ratio or rate and then solve a multistep problem; take a given ratio or rate and solve a multistep problem.
2. Solve single- and multistep problems involving percentages.	The student will solve a multistep problem to determine a percentage; calculate a percentage and then solve a multistep problem; take a given percentage and solve a multistep problem.
3. Solve single- and multistep problems involving measurement quantities, units, and unit conversion.	The student will solve a multistep problem to determine a unit rate; calculate a unit rate and then solve a multistep problem; solve a multistep problem to complete a unit conversion; solve a multistep problem to calculate density; use the concept of density to solve a multistep problem.
4. Given a scatterplot, use linear, quadratic, or exponential models to describe how the variables are related.	The student will, given a scatterplot, select the equation of a line or curve of best fit; interpret the line in the context of the situation; use the line or curve of best fit to make a prediction.
5. Use the relationship between two variables to investigate key features of the graph.	The student will make connections between the graphical representation of a relationship and properties of the graph by selecting the graph that represents the properties described; using the graph to identify a value or set of values.
6. Compare linear growth with exponential growth.	The student will infer the connection between two variables given a context in order to determine what type of model fits best.
7. Use two-way tables to summarize categorical data and relative frequencies, and calculate conditional probability.	The student will summarize categorical data or use categorical data to calculate conditional frequencies; conditional probabilities; association of variables; independence of events.
8. Make inferences about population parameters based on sample data.	The student will estimate a population parameter given the results from a random sample of the population. The sample statistics may mention confidence intervals and measurement error that the student should understand and make use of, but need not calculate.
9. Use statistics to investigate measures of center of data and analyze shape, center, and spread.	The student will calculate measures of center and/or spread for a given set of data or use given statistics to compare two separate sets of data. The measures of center that may be calculated include mean, median, and mode, and the measures of spread that may be calculated include range. When comparing two data sets, the student may investigate mean, median, mode, range, and/or standard deviation.
10. Evaluate reports to make inferences, justify conclusions, and determine appropriateness of data collection methods.	The student will evaluate reports to make inferences, justify conclusions, and determine appropriateness of data collection methods. The reports may consist of tables, graphs, and text summaries.

The redesigned SAT's Math Test has responded to the research evidence identifying what is essential for college readiness and success by focusing significantly on problem solving and data analysis: the ability to create a representation of a problem, consider the units involved, attend to the meaning of quantities, and know and use different properties of operations and objects. Problems in this category will require significant quantitative reasoning about ratios, rates, and proportional relationships and will place a premium on understanding and applying unit rate.

Interpreting and synthesizing data are widely applicable skills in postsecondary education and careers. In the redesigned SAT's Math Test, students will be expected to identify quantitative measures of center, the overall pattern, and any striking deviations from the overall pattern and spread in one or two different data sets. This includes recognizing the effects of outliers on the measures of center of a data set. In keeping with the need to stress widely applicable prerequisites, the redesigned SAT emphasizes applying core concepts and methods of statistics, rather than covering broadly a vast range of statistical techniques.

Finally, the redesigned SAT's Math Test emphasizes students' ability to apply math to solve problems in rich and varied contexts and features problems that require the application of problem solving and data analysis to solve problems in science, social studies, and career-related contexts.

## PASSPORT TO ADVANCED MATH: ANALYZING ADVANCED EXPRESSIONS

SAT PASSPORT TO ADVANCED MATH DOMAIN	
Content Dimension	Description
<b>Application</b>	
1. Create quadratic or exponential functions.	The student will create a quadratic or exponential function or equation that models a context. The equation will have rational coefficients and may require multiple steps to simplify or solve the equation.
2. Choose and produce equivalent forms of expressions to reveal and explain properties of a quantity.	The student will, given a context, determine the most suitable form of an expression or equation to reveal a particular trait.
<b>Procedural Skill and Fluency</b>	
3. Create equivalent expressions involving radicals and rational exponents.	The student will create equivalent expressions involving rational exponents and radicals, including simplifying or rewriting in other forms.
4. Create equivalent forms of expressions by using structure.	The student will create an equivalent form of an algebraic expression by using structure and fluency with operations.
5. Solve quadratic equations.	The student will solve a quadratic equation having rational coefficients. The equation can be presented in a wide range of forms to reward attending to algebraic structure and can require manipulation in order to solve.
6. Perform arithmetic operations on polynomials.	The student will add, subtract, and multiply polynomial expressions and simplify the result. The expressions will have rational coefficients.
7. Solve radical and rational equations in one variable, including examples where there are extraneous solutions.	The student will solve an equation in one variable that contains radicals or contains the variable in the denominator of a fraction. The equation will have rational coefficients, and the student may be required to identify when a resulting solution is extraneous.
8. Solve a system of equations consisting of one linear and one quadratic equation in two variables.	The student will solve a system of one linear equation and one quadratic equation. The equations will have rational coefficients.
9. Rewrite simple rational expressions.	The student will add, subtract, multiply, or divide two rational expressions or divide two polynomial expressions and simplify the result. The expressions will have rational coefficients.
<b>Conceptual Understanding</b>	
10. Interpret parts of nonlinear expressions in terms of their context.	The student will make connections between a context and the nonlinear equation that models the context to identify or describe the real-life meaning of a constant term, a variable, or a feature of the given equation.
11. Understand the relationship between zeros and factors of polynomials; use it to sketch graphs.	The student will use properties of factorable polynomials to solve conceptual problems relating to zeros, such as determining whether an expression is a factor of a polynomial based on other information provided.

**SAT PASSPORT TO ADVANCED MATH DOMAIN**

Content Dimension	Description
12. Understand a nonlinear relationship between two variables by making connections between their algebraic and graphical representations.	The student will select a graph corresponding to a given nonlinear equation, interpret graphs in the context of solving systems of equations, select a nonlinear equation corresponding to a given graph, determine the equation of a curve given a verbal description of a graph, determine key features of the graph of a linear function from its equation, or determine the impact to a graph of a change in the defining equation.
13. Use function notation, and interpret statements using function notation.	The student will use function notation to solve conceptual problems related to transformations and compositions of functions.
14. Use structure to isolate or identify a quantity of interest in an expression or isolate a quantity of interest in an equation.	The student will rearrange an equation or formula to isolate a single variable or a quantity of interest.

As a test that provides an entry point to postsecondary education and careers, the redesigned SAT's Math Test will include topics that are central to the ability of students to progress to later, more advanced mathematics. Problems in Passport to Advanced Math will cover topics that have great relevance and utility for college and career work.

Chief among these topics is the understanding of the structure of expressions and the ability to analyze, manipulate, and rewrite these expressions. This includes an understanding of the key parts of expressions, such as terms, factors, and coefficients, and the ability to interpret complicated expressions made up of these components. Students will be able to show their skill in rewriting expressions, identifying equivalent forms of expressions, and understanding the purpose of different forms.

This category also includes reasoning with more complex equations, including solving quadratic and higher-order equations in one variable and understanding the graphs of quadratic and higher-order functions. Finally, this category includes the ability to interpret and build functions, another skill crucial for success in later mathematics and scientific fields.

## ADDITIONAL TOPICS IN MATH

SAT ADDITIONAL TOPICS IN MATH DOMAIN	
Content Dimension	Description
<b>Application</b>	
1. Solve problems using volume formulas.	The student will use given information about figures, such as length of a side, area of a face, or volume of a solid, to calculate missing information. Any required volume formulas will be provided to students either on the formula sheet or within the question.
2. Use trigonometric ratios and the Pythagorean Theorem to solve applied problems involving right triangles.	The student will use information about triangle side lengths or angles presented in a context to calculate missing information using the Pythagorean theorem and/or trigonometric ratios.
<b>Procedural Skill and Fluency</b>	
3. Perform arithmetic operations on complex numbers.	The student will add, subtract, multiply, divide, and simplify complex numbers.
4. Convert between degrees and radians and use radians to determine arc lengths; use trigonometric functions of radian measure.	The student will convert between angle measures in degrees and radians in order to calculate arc lengths by recognizing the relationship between an angle measured in radians and an arc length, evaluating trigonometric functions of angles in radians.
5. Apply theorems about circles to find arc lengths, angle measures, chord lengths, and areas of sectors.	The student will use given information about circles and lines to calculate missing values for radius, diameter, chord length, angle, arc, and sector area.
<b>Conceptual Understanding</b>	
6. Use concepts and theorems about congruence and similarity to solve problems about lines, angles, and triangles.	The student will use theorems about triangles and intersecting lines to determine missing lengths and angle measures of triangles. The student may also be asked to provide a missing length or angle to satisfy a given theorem.
7. Use the relationship between similarity, right triangles, and trigonometric ratios; use the relationship between sine and cosine of complementary angles.	The student will use trigonometry and theorems about triangles and intersecting lines to determine missing lengths and angle measures of right triangles. The student may also be asked to provide a missing length or angle that would satisfy a given theorem.
8. Create or use an equation in two variables to solve a problem about a circle in the coordinate plane.	The student will create an equation or use properties of an equation of a circle to demonstrate or determine a property of the circle's graph.

While the overwhelming majority of problems on the redesigned SAT's Math Test fall into the first three domains, the test also addresses additional topics in high school math. In keeping with the approach described in Section II, patterns of selection for these are governed by evidence about their relevance to postsecondary education and work. The additional topics include essential geometric and trigonometric concepts and the Pythagorean Theorem, which become powerful methods of analysis and problem solving when connected to other math domains.

## SAMPLE QUESTIONS ILLUSTRATING DISTINCTIVE FEATURES OF THE REDESIGNED SAT'S MATH TEST

The following distinctive features of the redesigned SAT's Math Test are illustrated by sample questions that reflect the following:

- » An emphasis on mathematical reasoning over reasoning questions disconnected from the mathematics curriculum
- » A strong emphasis on both fluency and understanding
- » Richer applications, emphasizing career, science, and social studies applications
- » Item sets that allow for more than one question about a given scenario
- » A no-calculator portion

**REASONING ON THE REDESIGNED SAT'S MATH TEST WILL CONNECT MORE DIRECTLY TO ESSENTIAL SKILLS FOR COLLEGE READINESS THAT ARE PART OF A RIGOROUS HIGH SCHOOL CURRICULUM.**

To see what this shift means, consider the following question from the current SAT:<sup>1</sup>

Family	Number of Consecutive Nights
Jackson	10
Callan	5
Epstein	8
Liu	6
Benton	8

The table above shows the number of consecutive nights that each of five families stayed at a certain hotel during a 14-night period. If the Liu family's stay did not overlap with the Benton family's stay, which of the 14 nights could be a night on which only one of the five families stayed at the hotel?

- A) The 3rd
- B) The 5th
- C) The 6th
- D) The 8th
- E) The 10th

This question presents the student with a reasoning puzzle unrelated to the school mathematics curriculum. Being able to solve unfamiliar problems is valuable, but a test based entirely on this idea does not provide as much assurance that students have learned essential math skills and practices — nor does it reward students for their hard work in doing so.

The redesigned SAT's Math Test focuses on applied reasoning skills that are both essential for college readiness and taught in challenging high school math classrooms. This means that the questions will require reasoning and insight as they relate to important curricular skills such as looking for and making use of algebraic structure. In contrast to the question on the left, consider the following sample from the Heart of Algebra category:

EXAMPLE 1: Sample item from the redesigned SAT

If  $\frac{1}{2}x + \frac{1}{3}y = 4$ , what is the value of  $3x + 2y$ ?

A student may find the solution to this Heart of Algebra problem by noticing the structure of the given equation and seeing that multiplying both sides of the equation  $\frac{1}{2}x + \frac{1}{3}y = 4$  by 6 to clear fractions from the equation yields  $3x + 2y = 24$ .

<sup>1</sup> College Board, *Official SAT Practice Test 2013–14* (New York: Author, 2013).

## A STRONG EMPHASIS ON BOTH FLUENCY AND UNDERSTANDING

In *Adding It Up: Helping Children Learn Mathematics*, the National Research Council (NRC) identified procedural fluency and conceptual understanding as two of the five components of mathematical proficiency. The NRC calls for their inclusion in curricula, instructional materials, and assessments as they define what it means to learn math successfully.<sup>2</sup> As students cannot be ready for college and career without being mathematically proficient, the redesigned SAT assesses fluency with mathematical procedures and conceptual understanding with equal intensity.

The following two sample questions show some of the ways in which fluency and understanding are important on the redesigned SAT.

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### EXAMPLE 2

$$4x - y = 3y + 7$$

$$x + 8y = 4$$

Based on the system of equations above, what is the value of the product  $xy$ ?

- A)  $\frac{3}{2}$
- B)  $\frac{1}{4}$
- C)  $\frac{1}{2}$
- D)  $\frac{11}{9}$

Example 2, again from Heart of Algebra, rewards fluency in solving pairs of simultaneous linear equations. Rather than looking for a clever way of back solving the value of the product  $xy$  from the system, students can solve the system for the values of  $x$  and  $y$ , then simply multiply them to get choice C,  $\frac{1}{2}$ . Note that because the system is not given in standard form, this requires doing some additional algebra, further reinforcing the need for fluency.

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<sup>2</sup> National Research Council, *Adding It Up: Helping Children Learn Mathematics* (Washington, DC: The National Academies Press, 2001).

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**EXAMPLE 3**

The function  $f$  is defined by  $f(x) = 2x^3 + 3x^2 + cx + 8$ , where  $c$  is a constant. In the  $xy$ -plane, the graph of  $f$  intersects the  $x$ -axis at the three points  $(-4, 0)$ ,  $(\frac{1}{2}, 0)$ , and  $(p, 0)$ . What is the value of  $c$ ?

- A)  $-18$
- B)  $-2$
- C)  $2$
- D)  $10$

Example 3, from Passport to Advanced Math, assesses conceptual understanding of polynomials and their graphs. If a student understands these concepts and requires, for example, the point  $(-4, 0)$  to lie on the graph, this results in  $0 = 2(-4)^3 + 3(-4)^2 + c(-4) + 8$ . A student who looks for and makes use of structure will monitor the calculation at this point and recognize an equation that determines the desired value of  $c$ ,  $-18$ . Seeing that he or she is on the right track, the student will then perform the calculations required to solve for  $c$ .

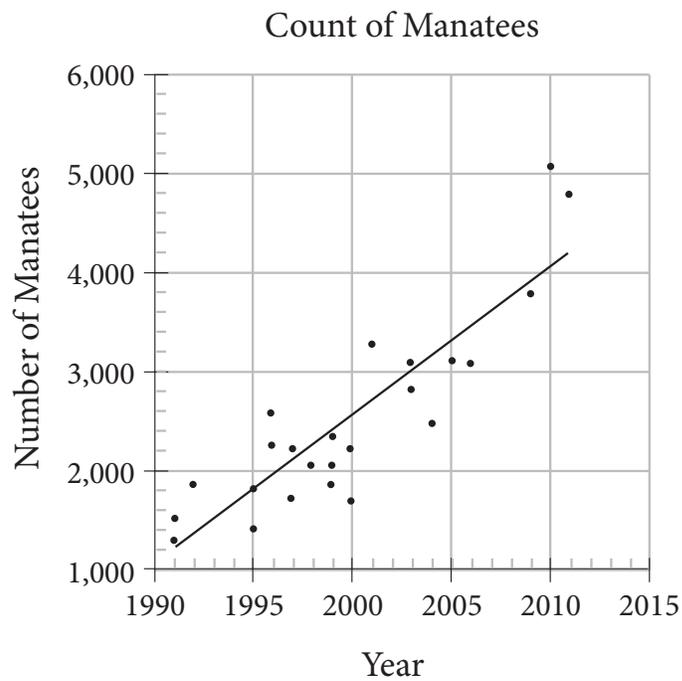
## **RICHER APPLICATIONS, EMPHASIZING CAREER, SCIENCE, AND SOCIAL STUDIES APPLICATIONS**

In response to evidence about essential prerequisites for college and career readiness and success, the redesigned SAT's Math Test requires students to apply their mathematics knowledge, skills, and understandings in challenging, authentic contexts. Students taking the Math Test will encounter a range of disciplines and will be asked to address real-world problems drawn from science, social studies, and careers and demonstrate a capacity for sustained reasoning over the multiple steps required to answer many of the questions on the exam. In these ways, the Math Test also rewards and incentivizes valuable work in the classroom.

Applications on the redesigned SAT's Math Test require students to demonstrate the ability to analyze a situation, determine the essential elements required to solve the problem, represent the problem mathematically, and carry out a solution. These applications often also require linking topics within the mathematics domain (e.g., functions and statistics) and across disciplines (e.g., math and science). Learning to model and problem solve is enhanced when students use the same mathematics (e.g., linear equations) to solve problems in different contexts (e.g., science, social studies, or careers).

Example 4 below is based on real-world methods (aerial observations of wintering spots, or synoptic counts) used by the U.S. Fish and Wildlife Service to count manatees, a type of sea mammal. This type of item is an excellent way of connecting linear functions to statistics. In this item, students are not required to model the line of best fit completely, but they are required to decontextualize the item to understand that they must compute the slope of the line of best fit to get the correct answer, 150.

## EXAMPLE 4

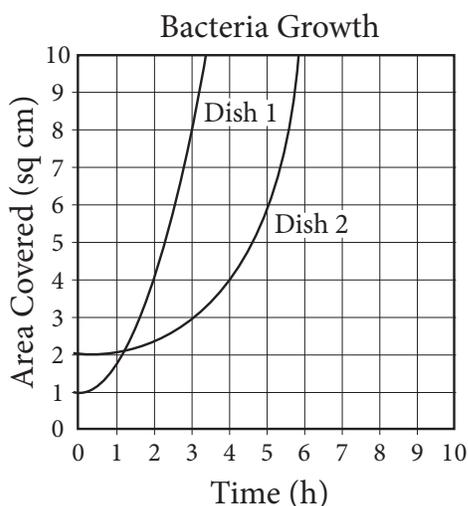


The scatterplot above shows counts of Florida manatees, a type of sea mammal, from 1991 to 2011. Based on the line of best fit to the data shown, which of the following values is closest to the average yearly increase in the number of manatees?

- A) 0.75
- B) 75
- C) 150
- D) 750

Example 5 below is another rich application item that uses a science context to make a connection across math domains (functions and statistics) and across subjects (math and science). In this item, students need to synthesize the information given in the graph and the prompt and determine which pieces of information in the graph will help provide them with a correct statement about the data.

## EXAMPLE 5



A researcher places two colonies of bacteria into two petri dishes that each have area 10 square centimeters. After the initial placement of the bacteria ( $t = 0$ ), the researcher measures and records the area covered by the bacteria in each dish every ten minutes. The data for each dish were fit by a smooth curve, as shown above, where each curve represents the area of a dish covered by bacteria as a function of time, in hours. Which of the following is a correct statement about the data above?

- A) At time  $t = 0$ , both dishes are 100% covered by bacteria.
- B) At time  $t = 0$ , bacteria covers 10% of Dish 1 and 20% of Dish 2.
- C) At time  $t = 0$ , Dish 2 is covered with 50% more bacteria than Dish 1.
- D) For the first hour, the area covered in Dish 2 is increasing at a higher average rate than the area covered in Dish 1.

## ITEM SETS THAT ALLOW FOR MORE THAN ONE QUESTION ABOUT A GIVEN SCENARIO

Asking more than one question about a given scenario allows students taking the redesigned SAT to do more sustained thinking and explore situations in greater depth. Students will encounter longer problems like these in their postsecondary work. By including item sets, the redesigned SAT rewards and incentivizes aligned, productive work in classrooms.

Item sets can be used to dig deeper into a student's understanding of a construct or to make connections to other domains. For example, one question from a set may ask about statistics and probability and the next may ask about the function that models the data. In the classroom, item sets manifest the connections between domains and provide opportunities for students to practice and extend their skills of abstraction, analysis, and communication.

Within this subset of questions, students will encounter:

- » career-related contexts, which could include scale drawings, estimation, unit rates, percentages, and proportional relationships;
- » problem sets that make use of these contexts, allowing multiple questions about a single stimulus;
- » real-life scenarios that will likely yield more complex solutions; and
- » real-life scenarios that might not be proportional; for instance, students may be asked to demonstrate their proficiency with scaling quantities that aren't proportional or with situations of diminishing returns and accelerated growth.

### ITEM SET:

In the classroom, item sets manifest the connections between different domains and provide opportunities for students to practice and extend their skills of abstraction, analysis, and communication. In the redesigned SAT, item sets allow the effective measurement of these skills and inspire productive practice in the classrooms.

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**EXAMPLE 6**

*(This is a student-produced response item set. Students grid in their answers, which are machine scored.)*

An international bank issues its Traveler credit cards worldwide. When a customer makes a purchase using a Traveler card in a currency different from the customer's home currency, the bank converts the purchase price at the daily foreign exchange rate and then charges a 4% fee on the converted cost.

Sara lives in the United States, but is on vacation in India. She used her Traveler card for a purchase that cost 602 rupees (Indian currency). The bank posted a charge of \$9.88 to her account that included the 4% fee.

**PART 1**

What foreign exchange rate, in Indian rupees per one U.S. dollar, did the bank use for Sara's charge? Round your answer to the nearest whole number.

**PART 2**

A bank in India sells a prepaid credit card worth 7,500 rupees. Sara can buy the prepaid card using dollars at the daily exchange rate with no fee, but she will lose any money left unspent on the prepaid card. What is the least number of the 7,500 rupees on the prepaid card Sara must spend for the prepaid card to be cheaper than charging all her purchases on the Traveler card? Round your answer to the nearest whole number of rupees.

**SOLUTION****PART 1**

\$9.88 represents the conversion of 602 rupees plus a 4% fee on the converted cost.

To calculate the original cost of the item in dollars,  $x$ :

$$\begin{aligned} 1.04x &= 9.88 \\ x &= 9.5 \end{aligned}$$

Since the original cost is \$9.50, to calculate the exchange rate  $r$ , in Indian rupees per one U.S. dollar:

$$\begin{aligned} 9.50 \text{ dollars} \times \frac{r \text{ rupees}}{1 \text{ dollar}} &= 602 \text{ rupees} \\ r &= \frac{602}{9.50} \\ &\approx 63 \text{ rupees} \end{aligned}$$

**PART 2**

Let  $d$  dollars be the cost of the 7,500-rupee prepaid card. This implies that the exchange rate on this particular day is  $\frac{d}{7,500}$  dollars per rupee. Suppose Sara's total purchases on the prepaid card were  $r$  rupees. The value of the  $r$  rupees in dollars is  $\left(\frac{d}{7,500}\right)r$  dollars. If Sara spent the  $r$  rupees on the Traveler card instead, she would be charged  $(1.04)\left(\frac{d}{7,500}\right)r$  dollars. To answer the question about how many rupees Sara must spend in order to make the Traveler card a cheaper option (in dollars) for spending the  $r$  rupees, we set up the inequality  $1.04\left(\frac{d}{7,500}\right)r \geq d$ . Rewriting both sides reveals  $1.04\left(\frac{r}{7,500}\right)d \geq (1)d$ , from which we can infer  $1.04\left(\frac{r}{7,500}\right) \geq 1$ . Dividing on both sides by 1.04 and multiplying on both sides by 7,500 finally yields  $r \geq 7,212$ . Hence the least number of rupees Sara must spend for the prepaid card to be cheaper than the Traveler card is 7,212.

Note that Example 6 is not a multiple-choice item. Responses are gridded in by students, which often allows for multiple correct responses and solution processes. Such items allow students to freely apply their critical thinking skills when planning and implementing a solution.

Example 6 is also an item set that includes two student-produced response questions. Student-produced response item set questions on the redesigned SAT measure the complex knowledge and skills that require students to deeply think through the solutions to problems. Set within a range of real-world contexts, these questions require students to make sense of problems and persevere in solving them; make connections between and among the different parts of a stimulus; plan a solution approach, as no scaffolding is provided to suggest a solution strategy; abstract, analyze, and refine an approach as needed; and produce and validate a response. These types of questions require the application of complex cognitive skills.

## A NO-CALCULATOR PORTION

The redesigned SAT's Math Test will contain two portions: one in which the student may use a calculator and another in which the student may not. The no-calculator portion allows the redesigned SAT to assess fluencies valued by postsecondary instructors and includes conceptual questions for which a calculator will not be helpful. Meanwhile, the calculator portion gives insight into students' capacity to use appropriate tools strategically. The calculator is a tool that students must use (or not use) judiciously.

The calculator portion of the test will include more complex modeling and reasoning questions to allow students to make computations more efficiently. However, this portion will also include questions in which the calculator could be a deterrent to expedience, thus assessing appropriate use of tools. For these types of questions, students who make use of structure or their ability to reason will reach the solution more rapidly than students who get bogged down using a calculator.

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**EXAMPLE 7**

What is one possible solution to the equation  $\frac{24}{x+1} - \frac{12}{x-1} = 1$ ?

Example 7, from the no-calculator portion of the test, requires students to look at the structure of the expression and find a way to rewrite it, again showing the link between fluency and mathematical practices. The student must transform the expression without a calculator, for example by multiplying both sides of the equation by a common denominator as a first step to find the solution. This leads to  $x = 5$  and  $x = 7$ , both of which should be checked in the original equation to ensure that they are not extraneous.

Additional example items showing the distinctive features of the redesigned SAT's Math Test within the four content categories can be found in Appendix B.

## Summary

The preceding discussion has presented an overview of the redesigned SAT's Math Test along with a discussion of some of the key features that make the Math Test distinctive both compared to the current SAT's math section and compared to other assessments within the field. As with the Evidence-Based Reading and Writing area of the redesigned SAT, we at the College Board are continuing our research and development of the redesigned SAT's Math Test. In doing so, we may find that we will need to make adjustments to our specifications as presented in this section (e.g., number of items, time limits, and scores). Any adjustments made, however, would be made only to more effectively serve the evidence-based features that are the focus of our development work on the Math Test. Our commitment, as we relate in Section V, includes communicating any and all changes widely and in a timely manner.