9.3 Trigonometric Functions of Any Angle

What You Will Learn

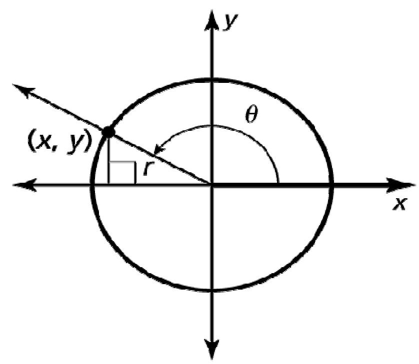
- Evaluate trigonometric functions of any angle.
- Find and use reference angles to evaluate trigonometric functions.

Trigonometric Functions of Any Angle

You can generalize the right-triangle definitions of trigonometric functions so that they apply to any angle in standard position

General Definitions of Trigonometric Functions

Let θ be an angle in standard position, and let (x, y) be the point where the terminal side of θ intersects the circle $x^2 + y^2 = r^2$. The six trigonometric functions of θ are defined as shown.



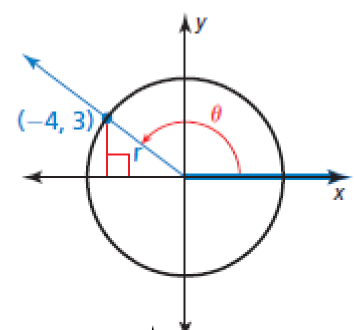
$\sin\theta = \frac{y}{r}$ $\qquad$ $\csc\theta = \frac{r}{y},\ y \neq 0$

$\cos\theta = \frac{x}{r}$ $\qquad$ $\sec\theta = \frac{r}{x},\ x \neq 0$

$\tan\theta = \frac{y}{x},\ x \neq 0$ $\qquad$ $\cot\theta = \frac{x}{y},\ y \neq 0$

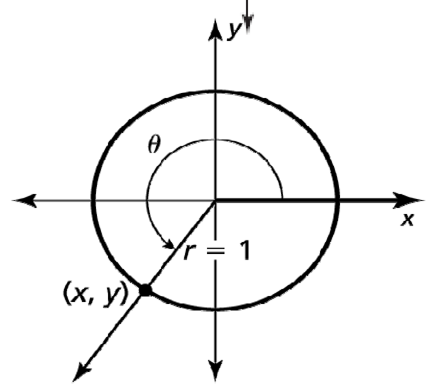
Example: **Evaluating Trigonometric Functions Given a Point**

Let (−4, 3) be a point on the terminal side of an angle θ in standard position. Evaluate the six trigonometric functions of θ



The Unit Circle

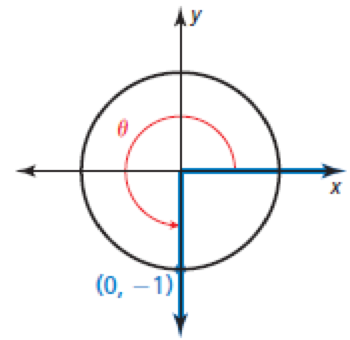
The circle $x^2 + y^2 = 1$, which has center (0, 0) and radius 1, is called the **unit circle**. The values of sin θ and cos θ are simply the y-coordinate and x-coordinate, respectively, of the point where the terminal side of θ intersects the unit circle.



$\sin\theta = \frac{y}{r} = \frac{y}{1} = y$ $\qquad$ $\cos\theta = \frac{x}{r} = \frac{x}{1} = x$

It is convenient to use the unit circle to find trigonometric functions of **quadrantal angles**. A quadrantal angle is an angle in standard position whose terminal side lies on an axis. The measure of a quadrantal angle is always a multiple of 90º, or $\frac{\pi}{2}$ radians.

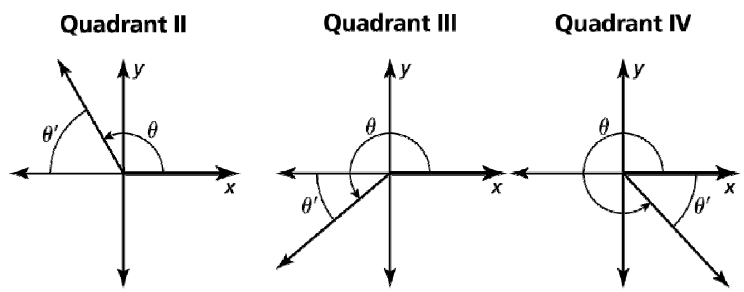
Example: **Using the Unit Circle**
Use the unit circle to evaluate the six trigonometric functions of $\theta = 270º$.



Example: Evaluate the six trigonometric functions of $\theta = 180º$.

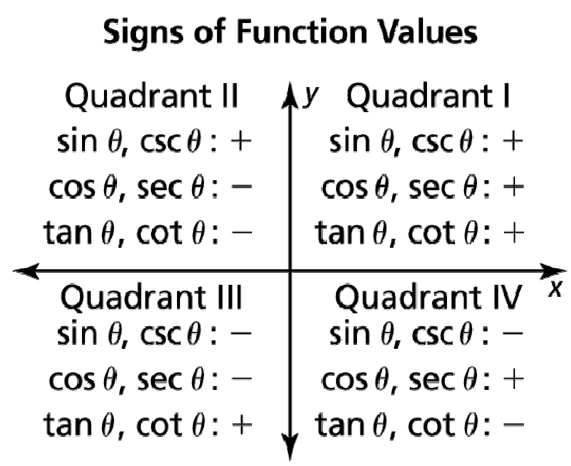
Reference Angle Relationships

Let θ be an angle in standard position. The **reference angle** for θ is the acute angle θ' formed by ***the terminal side of θ and the x-axis***. The relationship between θ and θ' is shown below for nonquadrantal angles θ such that $90° < \theta < 360°$ or, in radians, $\frac{\pi}{2} < \theta < 2\pi$.



Example: **Finding Reference Angles -** Find the reference angle for

(a) $\theta = \frac{5\pi}{3}$



(b) $\theta = -130º$.

Example: **Using Reference Angles to Evaluate Functions -** Evaluate

(a) tan(−240º)

(b) $\csc\frac{17\pi}{6}$

.

Sketch the angle. Then find its reference angle.

$\frac{15\pi}{4}$

Evaluate the function

$\sec\frac{11\pi}{4}$