9.1 Right Triangle Trigonometry

What You Will Learn

- Evaluate trigonometric functions of acute angles.
- Find unknown side lengths and angle measures of right triangles.
- Use trigonometric functions to solve real-life problems.

The Six Trigonometric Functions

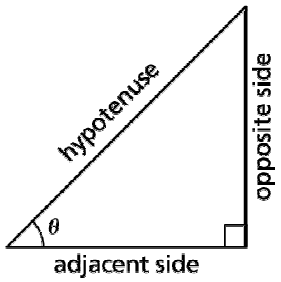
Consider a right triangle that has an acute angle θ (the Greek letter *theta*). The three sides of the triangle are the *hypotenuse*, the side *opposite* θ, and the side *adjacent* to θ.
Ratios of a right triangle's side lengths are used to define the six trigonometric functions: **sine**, **cosine**,**tangent**, **cosecant**, **secant**, and **cotangent**. These six functions are abbreviated sin, cos, tan, csc, sec, and cot, respectively.

Right Triangle Definitions of Trigonometric Functions

Let θ be an acute angle of a right triangle. The six trigonometric functions of θ are defined as shown.

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} \qquad \cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} \qquad \tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

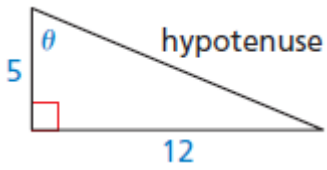
$$\csc\theta = \frac{\text{hypotenuse}}{\text{opposite}} \qquad \sec\theta = \frac{\text{hypotenuse}}{\text{adjacent}} \qquad \cot\theta = \frac{\text{adjacent}}{\text{opposite}}$$



The abbreviations *opp.*, *adj.*, and *hyp.* are often used to represent the side lengths of the right triangle. Note that the ratios in the second row are reciprocals of the ratios in the first row.

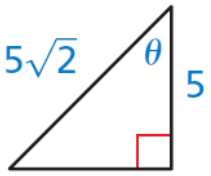
$$\csc\theta = \frac{1}{\sin\theta} \qquad \sec\theta = \frac{1}{\cos\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

Ex: **Evaluating Trigonometric Functions** *Evaluate the six trigonometric functions of the angle* θ



Ex: **Evaluating Trigonometric Functions** In a right triangle, θ is an acute angle and $\sin\theta = \frac{4}{7}$. Evaluate the other five trigonometric functions of θ.

Evaluate the six trigonometric functions of the angle θ.



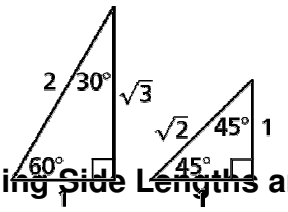
In a right tri., θ is an acute angle and $\cos\theta = \frac{7}{10}$. Evaluate the other five trigonometric functions of θ.

The angles 30, 45, and 60 occur frequently in trigonometry. You can use the trigonometric values for these angles to find unknown side lengths in special right triangles.

Trigonometric Values for Special Angles

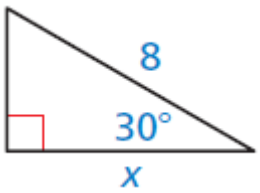
The table gives the values of the six trigonometric functions for the angles 30°, 45°, and 60°. You can obtain these values from the triangles shown.

θ	sin θ	cos θ	tan θ	csc θ	sec θ	cot θ
30°	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$	2	$\frac{2\sqrt{3}}{3}$	$\sqrt{3}$
45°	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1	$\sqrt{2}$	$\sqrt{2}$	1
60°	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{2\sqrt{3}}{3}$	2	$\frac{\sqrt{3}}{3}$



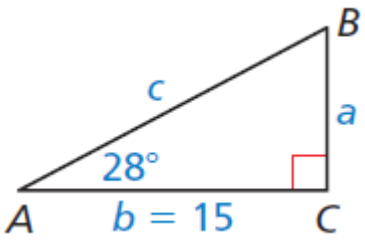
Finding Side Lengths and Angle Measures

Example: **Finding an Unknown Side Length -** *Find the value of x for the right triangle.*



Finding all unknown side lengths and angle measures of a triangle is called *solving the triangle*. Solving right triangles that have acute angles other than 30°, 45°, and 60° may require the use of a calculator. Be sure the calculator is set in *degree* mode.

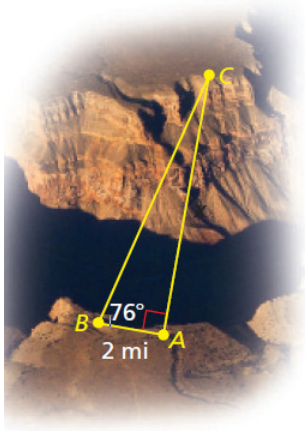
Example: **Using a Calculator to Solve a Right Triangle -** *Solve* $\triangle ABC$



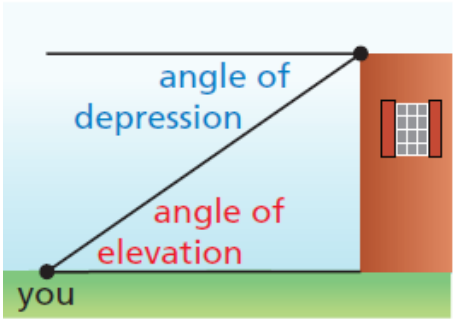
Solving Real-Life Problems

Using Indirect Measurement

You are hiking near a canyon. While standing at *A*, you measure an angle of 90º between *B* and *C*, as shown. You then walk to *B* and measure an angle of 76° between *A* and *C*. The distance between *A* and *B* is about 2 miles. How wide is the canyon between *A* and *C*?



If you look at a point above you, such as the top of a building, the angle that your line of sight makes with a line parallel to the ground is called the *angle of elevation*. At the top of the building, the angle between a line parallel to the ground and your line of sight is called the *angle of depression*. These two angles have the same measure.



Using an Angle of Elevation

A parasailer is attached to a boat with a rope 72 feet long. The angle of elevation from the boat to the parasailer is 28° . Estimate the parasailer's height above the boat.