7.3 Multiplying and Dividing Rational Expressions

What You Will Learn

- Simplify rational expressions.
- Multiply rational expressions.
- Divide rational expressions.

Simplifying Rational Expressions

A **rational expression** is a fraction whose numerator and denominator are nonzero polynomials. The *domain* of a rational expression excludes values that make the denominator zero. A rational expression is in **simplified form** when its numerator and denominator have no common factors (other than ± 1). ***Think simplifying fractions!***

Core Concept

Simplifying Rational Expressions

Let a, b, and c be expressions with $b \neq 0$ and $c \neq 0$.

Property $\frac{a\cancel{c}}{b\cancel{c}} = \frac{a}{b}$ Divide out common factor c.

Examples $\frac{15}{65} = \frac{3 \cdot \cancel{5}}{13 \cdot \cancel{5}} = \frac{3}{13}$ Divide out common factor 5.

$\frac{4\cancel{(x+3)}}{(x+3)\cancel{(x+3)}} = \frac{4}{x+3}$ Divide out common factor $x + 3$.

Simplifying a rational expression usually requires two steps. First, factor the numerator and denominator. Then, divide out any factors that are common to both the numerator and denominator. Here is an example:

$$\frac{x^2 + 7x}{x^2} = \frac{x(x+7)}{x \cdot x} = \frac{x+7}{x}$$

Example: Simplifying a Rational Expression – *Simplify* $\frac{x^2-4x-12}{x^2-4}$

COMMON ERROR

Do not divide out variable terms that are not factors.

$\frac{x-6}{x-2} \neq \frac{-6}{-2}$

Simplify the rational expression, if possible.

1. $\frac{2(x+1)}{(x+1)(x+3)}$ 2. $\frac{x+4}{x^2-16}$ 3. $\frac{4}{x(x+2)}$ 4. $\frac{x^2-2x-3}{x^2-x-6}$

Multiplying Rational Expressions

The rule for multiplying rational expressions is the same as the rule for multiplying numerical fractions: multiply numerators, multiply denominators, and write the new fraction in simplified form. Similarly to rational numbers, rational expressions are closed under multiplication.

Core Concept

Multiplying Rational Expressions

Let a, b, c, and d be expressions with $b \neq 0$ and $d \neq 0$.

Property $\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$ Simplify $\frac{ac}{bd}$ if possible.

Example $\frac{5x^2}{2xy^2} \cdot \frac{6xy^3}{10y} = \frac{30x^3y^3}{20xy^3} = \frac{\cancel{10} \cdot 3 \cdot \cancel{x} \cdot x^2 \cdot \cancel{y^3}}{\cancel{10} \cdot 2 \cdot \cancel{x} \cdot \cancel{y^3}} = \frac{3x^2}{2}$, $x \neq 0, y \neq 0$

Example: Multiplying Rational Expressions - *Find the product* $\frac{8x^3y}{2xy^2} \cdot \frac{7x^4y^3}{4y}$

Example: Multiplying Rational Expressions - *Find the product* $\frac{3x - 3x^2}{x^2 + 4x - 5} \cdot \frac{x^2 + x - 20}{3x}$.

Example: Multiplying a Rational Expression by a Polynomial

Find the product $\frac{x + 2}{x^3 - 27} \cdot (x^2 + 3x + 9)$.

Dividing Rational Expressions
To divide one rational expression by another, multiply the first rational expression by the reciprocal of the second rational expression. Rational expressions are closed under nonzero division.

Core Concept

Dividing Rational Expressions

Let a, b, c, and d be expressions with $b \neq 0$, $c \neq 0$, and $d \neq 0$.

Property $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{ad}{bc}$ Simplify $\frac{ad}{bc}$ if possible.

Example $\frac{7}{x+1} \div \frac{x+2}{2x-3} = \frac{7}{x+1} \cdot \frac{2x-3}{x+2} = \frac{7(2x-3)}{(x+1)(x+2)}, x \neq \frac{3}{2}$

Example: **Dividing Rational Expressions**

Find the quotient $\frac{7x}{2x-10} \div \frac{x^2-6x}{x^2-11x+30}$.

Example: **Dividing a Rational Expression by a Polynomial**

Find the quotient $\frac{6x^2+x-15}{4x^2} \div (3x^2+5x)$.

Find the product.

5. $\dfrac{3x^5y^2}{8xy} \cdot \dfrac{6xy^2}{9x^3y}$

6. $\dfrac{2x^2 - 10x}{x^2 - 25} \cdot \dfrac{x + 3}{2x^2}$

7. $\dfrac{x + 5}{x^3 - 1} \cdot (x^2 + x + 1)$

Find the quotient.

8. $\dfrac{4x}{5x - 20} \div \dfrac{x^2 - 2x}{x^2 - 6x + 8}$

9. $\dfrac{2x^2 + 3x - 5}{6x} \div (2x^2 + 5x)$